Comparison of Routing and Wavelength Assignment Algorithms in WDM Networks

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***Abstract* - We design and implement various algorithms for solving the static RWA problem with the objective of minimizing the maximum number of requested wavelengths based on LP relaxation formulations. We present a link formulation, a path formulation and a heuristic that breaks the problem in the two constituent subproblems and solves them individually and sequentially. The flow cost functions that are used in these formulations result in providing integer optimal solutions despite the absence of integrality constraints for a large subset of RWA input instances, while also minimizing the total number of used wavelengths. We present a random perturbation technique that is shown to increase the number of instances for which we find integer solutions, and we also present appropriate iterative fixing and rounding methods to be used when the algorithms do not yield integer solutions. We comment on the number of variables and constraints these formulations require and perform extensive simulations to compare their performance to that of a typical min-max congestion formulation.**

***Index Terms*— Routing and Wavelength Assignment, LP relaxation, piecewise linear costs, perturbation**

I. INTRODUCTION

Optical networks rely on *wavelength division multiplexing* (WDM) to efficiently exploit the massive available bandwidth. WDM enables different connections to be established concurrently through a common set of fibers, subject to the *distinct wavelength assignment constraint*; that is, the connections sharing a fiber must occupy separate wavelengths.

The most common architecture utilized for establishing communication in WDM optical networks is *wavelength routing* [1], where messages are transmitted through *lightpaths*; that is, all-optical WDM channels, that may span multiple consecutive fibers. In the absence of wavelength conversion, a lightpath must be assigned a common wavelength on each link it traverses; this restriction is referred to as the *wavelength continuity constraint*. Given a set of requested connections, the problem of setting up lightpaths by routing and assigning wavelengths to them, so as to minimize the network resources used or maximize the traffic served, is called the *routing and wavelength assignment* (RWA) problem.

The RWA problem is usually considered under two alternative traffic models. *Static* (or *Offline*) *Lightpath Establishment* addresses the case where the set of connections is known in advance. *Dynamic* (or *Online*) *Lightpath Establishment* considers the case where connection requests arrive randomly, and are served on a one-by-one basis. In this study we will focus on the static RWA problem.

Static RWA is known to be an NP-hard optimization problem since it is considered as a special case of the integer multicommodity flow (MCF) problem with additional constraints and can be formulated as an *integer linear program* (ILP). A review on static RWA algorithms can be found in [2].

The authors in [3] cope with the problem of minimizing the number of wavelengths to serve a given set of connections, what is usually called the Min-RWA problem. They give an ILP formulation and propose ways to solve it in an efficient manner for large networks. They break the problem into the routing and the wavelength assignment subproblems and solve them sequentially by employing a randomized rounding technique on a multicommodity flow (MCF) formulation, and a smallest last graph coloring algorithm, respectively. Simulations experiments of networks up to 100 nodes that have, however, high connectivity degree and small traffic load are reported. An LP formulation for Min-RWA recently proposed in [4] was shown to produce optimal integer solutions (without rounding) for a great fraction of RWA instances, despite the absence of integrality constraints. The formulation uses a specifically designed cost function that tries to avoid link congestion while also being piecewise linear so as to exhibit good integrality performance when the Simplex algorithm is used.

The dual problem, usually referred to as the Max-RWA problem, objects to maximize the number of connections established, while the traffic characteristics are a-priori given and the network resources (number of available wavelengths) are constrained. The authors in [5] give the ILP formulation for the Max-RWA, use the LP relaxation to obtain an upper bound on the traffic that can be routed and use this bound as a reference metric to evaluate a shortest-path RWA heuristic algorithm. The same problem is examined in [6], where the authors prove that the upper bound can be computed exactly by solving a significantly simplified LP that considers only one wavelength. In [7] the authors provide two ILP formulations for the Max-RWA problem that are based on multicommodity flow (MCF) formulations. To obtain results for large networks they describe two algorithms that are based on the relaxed linear formulations combined with proper rounding techniques.

In general, there are two classes of formulations, (i) those with link or node variables and (ii) those with path variables. [3], [5] and [7] follow the first approach, while [4] and [6] follow the second. A comparison of general types of link and path formulations is presented in [8]. The authors also provide proofs for the optimality that the relaxed LP solutions of the link and path formulations can achieve.

Routing and the wavelength assignment problems are often solved sequentially rather than jointly in order to make the problem more computationally tractable [3]. Various efficient heuristics have been lately developed for both routing and wavelength assignment, that may be combined and produce solutions for the joint RWA problem [9]-[12]. However, such decomposition suffers from the drawback that the optimal

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solution of the (joint) RWA problem might not be included in the solutions provided by the decomposed algorithms.

In this paper we present various algorithms for solving the static RWA problem, while minimizing the maximum number of requested wavelengths, based on LP relaxation formulations. Our work is based on [4] which gives a link formulation and a general form of a piecewise linear cost function. In this paper we extend [4] and present a path and a decomposed heuristic formulation, and three candidate cost functions. We also present a random perturbation technique that makes these formulations yield more integer solutions, and iterative fixing and rounding methods to be used when the algorithms do not yield integer solutions. We compare all these variations to a typical min-max congestion formulation that is widely used in the literature, and we also obtain results for an ILP execution to be used as a reference. We use the maximum number of wavelengths, the running time and the percentage of instances for which we obtained integer solutions as performance metrics. Our results indicate that the proposed algorithms outperform the min-max formulation and approximate the performance of an ILP execution with respect to the maximum number of required wavelengths, while decreasing the execution time by several orders of magnitude depending on the network topology and the traffic load.

The rest of this paper is organized as follows. In Section II we define our RWA algorithms. More specifically, the link and path formulations are given in Section II.A, and the decomposed algorithm in Section II.B. In Section II.C we present flow cost functions that can be used with these formulation, and the min-max congestion formulation that is widely used in the literature. In Section II.D we comment on the variables and constraints that these formulations require. In Section II.E we present the mechanisms for handling non integer solutions. In Section III we report our performance results and Section IV concludes the paper.

II. Definitions of RWA algorithms

A fixed network topology is represented by a connected graph $G=(V,E)$. V denotes the set of nodes, which we assume not to be equipped with wavelength conversion capabilities. E denotes the set of point-to-point single-fiber links. Each fiber is able to support a set $C=\{1,2,\ldots,W\}$ of W distinct wavelengths. The static version of RWA assumes an a-priori known traffic scenario given in the form of a matrix of nonnegative integers $\mathbf{\Lambda}$, called the traffic matrix.

The algorithm is given a specific RWA instance; that is, a network topology, its nodes' and links' characteristics, the set of wavelengths that can be used, and a static traffic scenario. It returns the instance solution, in the form of routed lightpaths and assigned wavelengths, and the number of wavelengths required to route all the connections, as well as the blocking probability that accounts for requests that are not served for the given number of wavelengths W.

We propose and evaluate two algorithms that jointly address the RWA problem (Section II.A) and an algorithm that decomposes the problem in its constituent subproblems, namely routing and wavelength assignment, and solves these subproblems individually and sequentially (Section II.B).

A. Joint RWA algorithms: Link and Path formulations

The joint RWA algorithms consist of four phases.

Phase 1: In this phase, k candidate paths are identified for serving each requested connection. These are selected by employing a variation of the k-shortest path algorithm. After a subset P_{sd} of candidate paths for each commodity pair $s-d$ is computed, the total set of computed paths, $P = \bigcup_{s-d} P_{sd}$, is inserted to the next phase. The time complexity of the preprocessing phase is clearly polynomial.

Phase 2: Taking into account the network characteristics (topology, type of links, number of available wavelengths), the traffic matrix, and the set of paths identified in Phase 1, Phase 2 formulates the given RWA instance as an LP problem. The two LP formulations used are presented in the following two subsections. The corresponding LP is solved using Simplex, which is generally considered efficient for the great majority of possible inputs. If the instance is feasible and the solutions are integer, the algorithm terminates by returning the optimal solution in the form of routed lightpaths and assigned wavelengths, and throughput equal to 1. If the instance is feasible but the solutions are fractional we proceed to Phase 3. If the instance is infeasible, meaning that it cannot be solved with the given number of wavelengths, we go to Phase 4.

Phase 3: In case of a fractional (non-integer) solution, the third phase involves iterative fixing and rounding methods, as presented in Section II.E, to obtain integer solutions. The maximum number of iterations is the number of variables, which is polynomial on the size of the input. As later explained, rounding can increase the number of required wavelengths, in which case the surplus wavelengths have to be removed. The removed wavelengths are those occupied by the minimum number of lightpaths, so as to block a small number of connections. The algorithm terminates and outputs the routed lightpaths and assigned wavelengths, along with the throughput that can be less than 1 due to the rounding process.

Phase 4: This phase is used when the LP instance is infeasible for the given number of wavelengths W. Infeasibility is overcome by iteratively increasing the number of available wavelengths and re-executing Phases 2 and 3 until a feasible solution is found. At the end of Phase 4 we have to select which connections should be blocked so as to reduce the number of wavelengths to the given set C. The algorithm terminates and outputs the routed lightpaths and assigned wavelengths, along with the throughput, which is less than 1.

1. Link formulation

The formulation presented in this section uses link-related variables.

Parameters:

- $s,d \in V$: network nodes
- $w \in C$: an available wavelength
- $l,l' \in E$: network links
- $p \in P_{sd} \subset P$: a candidate path

Constant:

- Λ_{sd} : the number of requested connections from s to d

Variables:

- λ_{plw}: an indicator variable, equal to 1 if path p occupies wavelength w on link l, and equal to 0 otherwise
- F_l : the flow cost value of link l. F_l is a function of w_l which is the number of lightpaths traversing link l

LINK FORMULATION

$$\text{minimize: } \sum_l F_l$$

subject to the following constraints:

- Distinct wavelength assignment constraint: $\sum_{p} \lambda_{plw} \le 1$, for all $l \in L$, for all $w \in C$.
- Wavelength continuity constraint: $\lambda_{plw} = \lambda_{pl'w}$, for all sd, where l and l' are consecutive links in path p.
- Incoming traffic constraint: $\sum_{p \in P_{sd}} \sum_{w} \lambda_{plw} = \Lambda_{sd}$, for all sd, when l is the first link on p
- Flow cost per link l: $F_l \ge f(w_l) = f\left(\sum_{p \in P} \sum_{w} \lambda_{plw}\right)$

Various flow cost functions f that can be used with this formulation are presented in Section II.C.

2. Path formulation

The formulation presented in this section uses path-related variables. In particular, λ_{pw} is an indicator variable, equal to 1 if path p occupies wavelength w, and equal to 0 otherwise. Also, as before, F_l variable denotes the flow cost value of link l.

PATH FORMULATION

minimize: $\sum_{l} F_l$

subject to the following constraints:

- Distinct wavelength assignment constraint: $\sum_{p|l \in p} \lambda_{pw} \le 1$, for all $l \in L$, for all $w \in C$.
- Incoming traffic constraint: $\sum_{p \in P_{sd}} \sum_{w} \lambda_{pw} = \Lambda_{sd}$, for all sd
- Flow cost per link l: $F_l \ge f(w_l) = f\left(\sum_{p|l \in p} \sum_{w} \lambda_{pw}\right)$

Comparing this formulation to the link formulation we have to note that the wavelength continuity constraints is implicitly taken into account by the definition of the λ_{pw} variables.

Various flow cost functions f that can be used with this formulation are presented in Section II.C.

B. Decomposed algorithm: Individual consideration of routing and wavelength assignment subproblems

The algorithm presented in this section breaks the problem to (i) the routing and (ii) the wavelength assignment subproblems and addresses each problem separately and sequentially. The proposed algorithm is quite similar to the algorithm presented in [3]. Note that by breaking the problem, the joint optimum of the RWA problem might not be found.

The decomposed algorithm consists again of 4 phases. Contrary to the previous algorithms we don't use a k-shortest path algorithm but in the first phase we formulate the routing subproblem as an LP multicommodity flow problem. Fractional and infeasible solutions are handled as phases 3 and 4 of Section II.A (now phases 2 and 3, respectively). Phase 4 incorporates a graph coloring algorithm to perform the wavelength assignment. The graph coloring algorithm that we employ is the smallest last (SL) algorithm [3]. Note that in the MCF formulation we do not use candidate paths for each connection request but we provide flow constraints instead. The solution of the routing problem that is inserted in phase 4 is a path to serve each connection, or more precisely the links that comprise each path. In particular, $\lambda_{l\,sd}$ is an indicator variable, equal to 1 if link l is used by connection sd, or equal to 0 otherwise. Also, as before, F_l variable denotes the flow cost value of link l.

MULTICOMMODITY FLOW FORMULATION

minimize: $\sum_{l} F_l$

subject to the following constraints:

- Incoming traffic constraint: $\sum_{l \text{ starting from } s} \lambda_{l\,sd} = \Lambda_{sd}$, for all sd,
- Outgoing traffic constraint: $\sum_{l \text{ ending at } d} \lambda_{l\,sd} = \Lambda_{sd}$, for all sd,
- Flow continuity constraint: $\sum_{l \text{ ending at } n} \lambda_{l\,sd} = \sum_{l' \text{ starting from } n} \lambda_{l'\,sd}$, for all sd, for all $n \in V \neq s,d$
- Flow cost per link, $F_l \ge f(w_l) = f\left(\sum_{sd} \lambda_{l\,sd}\right)$

Various flow cost functions f that can be used with this formulation are presented in Section II.C.

C. Cost Functions

Irrespective of the formulation (link, path, or decomposed) we use a flow cost function F_l to express the amount of congestion on each link. To do so we express F_l as a function $f(w_l)$, where w_l is the number of lightpaths crossing link l.

The chosen function f should be a properly increasing function of w_l. Also f should be chosen so as to imply a greater amount of 'undesirability', when a link becomes highly congested. That is f should be convex. Moreover, f should be more rapidly increasing at higher levels of congestion. That is $f(w_l+dw_l)/f(w_l)$ should be also increasing. The argument is that we prefer, in terms of network performance, few low-congested links to be added one flow unit, than a single link to be totally congested, since in the latter case, a significant number of candidate paths is probably blocked and routing options are limited. Remember that we want to minimize the maximum number of wavelengths to serve the connections.

We utilize the following flow cost functions:

i) linear: $F_l=f(w_l)=w_l$
ii) square: $F_l=f(w_l)= w_l^{\,2}$
iii) exponential: $F_l=f(w_l)= 2^{w_l/(W+1-w_l)}$,

where W is the maximum number of available wavelengths of a link. Note that the maximum value of the third function is $f(W)=2^W$, which can give acceptable resolution up to W=32, given that the integers are mapped in the system by 4 bytes.

Note that (i) is an increasing function, (ii) is an increasing and convex function, and (iii) is an increasing and convex function with $f(w_l+dw_l)/f(w_l)$ also increasing convex.

Functions (ii) and (iii) that are nonlinear are inserted to the LP formulations in the approximate form of a piecewise linear function; i.e., a continuous non-smooth function, that consists of W consecutive linear parts (Figure 1).

Inserting a sum of such piecewise linear functions to the LP objective, results in the identification of integer optimal solutions by Simplex, in most cases. This is because the vertices of the polyhedron defined by the constraints tend to correspond to the corner points of the piecewise linear function and thus consist also of integer components. Since the Simplex algorithm moves from vertex to vertex of that polyhedron there is a higher probability of obtaining integer solutions than other using methods.

1. Typical min-max congestion cost function

For this kind of optimization we alter slightly the formulations presented in Sections II.A and II.B.

The objective is to minimize F_{max},
and the flow cost constraints become: $F_{\max} \geq w_l$.

Note that this is the actual objective that we want to minimize, since F_{max} defines the maximum number of wavelengths among all links needed to serve the connections. The flow cost functions presented in the previous section try to approximate F_{max} while also being continuous and piecewise linear, so as to exhibit a good integrality performance when the Simplex algorithm is used. Note that the mix-max congestion cost function is the one used in the majority of the formulations found in the literature [2].

D. Variables and constraints

In Table I we analyze the algorithms that were presented in the previous sections with respect to the number of variables that they utilize and the number of constraints that they require. We can see the usefulness of using k-shortest paths in our path and link formulations, since by controlling k we can control the number of variables and constraints. Formulations that do not depend on k-shortest paths, like [7] and our MCF formulation, are more sensitive to the used topology (number of nodes, connectivity degree). The link formulation has h_{avg} more variables than the path formulation, and more constraints. MCF formulation's variables do not depend on the number of wavelengths W, but on the number of links L.

E. Handling non integer solutions

Although the piecewise linear cost functions presented in Section II.C are designed so as to have good integrality, there are still cases that the solution is not integer. To obtain integer solutions we employ the following methods.

1. Random perturbation techniques

In the general multicommodity problem, [13], given a fractional solution, a flow that is served by more than one paths has equal sum of first derivates over the links of those paths and also these paths are of equal length. Since the objective function that we utilize sums the cost flows of the links that comprise a path, a request that is served by more than one lightpaths has equal sums of first derivates over the links of these paths. Note that the derivative of a lightpath on a specific link is given by the slope of the linear or piecewise linear cost function that we utilize.

To avoid such cases, we multiply the slopes of each variable on each link with a random number that is close to 1. More specifically, we generate random numbers that differ to 1 in

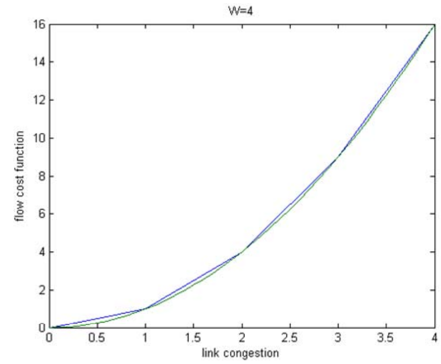

Fig. 1: The flow cost function $F_l=f(w_l)=w_l^2$ (curved line) and the corresponding piecewise linear function.

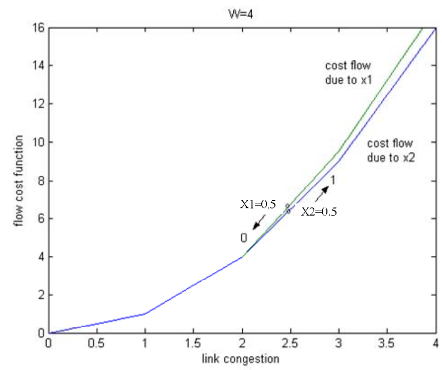

Fig. 2: Random perturbation mechanism. Let two lightpaths equally serve a connection request (variables $x_1=0.5$ and $x_2=0.5$). If the derivatives of these variables on a link are not equal, one lightpath is preferably selected.

the sixth decimal digit and multiply with them the slopes of each variable. In this way, the cases that two variables have equal derivates over the links that comprise a path are reduced, and thus we obtain more integer solutions (Figure 2). Note that altering the derivates of the variables is a tricky process. Such technique can make the Simplex algorithm perform more steps and search more feasible solutions until it finally finds the optimum, since a variable that has smaller derivate is aided and might be searched first by Simplex.

2. Iterative methods to increase the number of integer variables

If the LP execution gives (some) non-integer solutions we employ the following iterative fixing and rounding methods.

Fixing variables

We treat those solution values that are integer as final, and solve the reduced problem for the remaining variables. This is equivalent to making these variables constants and solve the initial LP problem with these additional equality constraints. In the latter case we do not need to create a new tableau, which can be time consuming. Fixing variables does not change the

TABLE I
NUMBER OF VARIABLES AND CONSTRAINTS RWA FORMULATIONS

Formulation	Cost Functions	Number of Variables	Number of constraints	
			=	≤
Link	(i) Linear	$k \cdot p \cdot N^2 \cdot h_{avg} \cdot W+L$	$(p \cdot N^2)_1+(k \cdot p \cdot N^2 \cdot h_{avg} \cdot W)_3$	$(L \cdot W)_5+(L)_6$
	(ii),(iii) Piecewise linear			$(L \cdot W)_5+(L \cdot W)_6$
	(iv) Min-max	$k \cdot p \cdot N^2 \cdot h_{avg} \cdot W+1$		$(L \cdot W)_5+(L)_6$
Path	(i) Linear	$k \cdot p \cdot N^2 \cdot W+L$	$(p \cdot N^2)_1$	$(L \cdot W)_5+(L)_6$
	(ii),(iii) Piecewise linear			$(L \cdot W)_5+(L \cdot W)_6$
	(iv) Min-max	$k \cdot p \cdot N^2 \cdot W+1$		$(L \cdot W)_5+(L)_6$
Multi-commodity flow (MCF)	(i) Linear	$p \cdot N^2 \cdot L+L$	$(p \cdot N^2)_1+(p \cdot N^2)_2+(p \cdot N^3)_4$	$(L)_6$
	(ii),(iii) Piecewise linear			$(L \cdot W)_6$
	(iv) Min-max	$p \cdot N^2 \cdot L+1$		$(L)_6$

N: number of nodes
W: number of wavelengths
L: number of links
h_{avg}: average number of links per path
k: number of shortest paths for each connection
p: load (percentage of total connections)

Constraints:
1: incoming traffic constraints
2: outgoing traffic constraints
3: wavelength continuity constraints
4: flow continuity constraints
5: distinct wavelength assignment constraints
6: cost function constraints

objective cost returned by the LP, so we move with each fixing from the previous solution to a solution with equal or more integers with the same cost. Thus, if after successive "fixings" we reach an all-integer solution we are sure that it is an optimal solution. On the other hand, fixing variables is not guaranteed to return an integer solution if one exists, since the integer solution might consist of different integer variables than the ones gradually fixed by this process. When we reach a point that the process of fixing does not increase the integrality of the solution, we proceed to the rounding process.

Rounding a single variable

Having a set of non-integer solutions we have to choose one variable to round. Choosing this variable is not a trivial task, since this might result in an increase in the objective. Thus, we want to round the variable that results in the smallest (or none) increase of the objective. This is the variable whose derivate is smaller with respect to the used objective function. Instead of finding this variable, something that would require additional calculations, we round a single variable, the one closest to 1, and continue solving the reduced LP problem.

Rounding is inevitable when there is no integer solution with the same objective cost as the ILP relaxation of the RWA instance. While fixing variables helps us move to more integer solutions with the same objective, rounding helps us move to a higher objective and search for an integer solution there. Thus, if even a single variable is rounded we are not sure anymore that we will find an optimal solution.

IV. PERFORMANCE RESULTS

To evaluate the performance of proposed algorithms we performed a large number of simulation experiments. All experiments were executed in MATLAB. For LP and ILP solving, we used the GLPK-4.16 MATLAB library [14]. We assumed the NSFnet topology that has 14 nodes and 21 edges (we assumed 42 directed links). We examined all the possible combinations of formulations and flow cost functions, presented in Sections II.A, II.B, II,C, and compared their performance to that of the typical min-max flow congestion formulation presented in Section II.C.1.

To have a reference point we also executed the same experiments using the path formulation and the exponential cost function (third function of Section II.C) with an ILP branch-and-bound algorithm. Note that the ILP execution of the path formulation with the min-max flow cost function would be the ultimate criterion since it is bound to give the optimal solution with respect to the maximum number of wavelengths required to solve a specific instance. However, using min-max ILP we were not able to obtain a solution proven to be optimum within an acceptable time (e.g. 4 hours) for some input instances, and especially for high traffic loads. This observation strengthens our belief that the proposed linear and piecewise linear cost functions are more appropriate and improve the performance of the RWA algorithms even when ILP is considered.

The results were averaged over 100 experiments corresponding to different random static traffic instances of a given traffic load. More specifically we have performed experiments for loads equal to 0.5, 0.75 of the total number of connections. To fairly compare the algorithms we have produced the same 100 instances.

To evaluate the performance of the algorithms we used the following metrics:

(a) The maximum of used wavelengths averaged over all experiments
(b) The sum of used wavelengths averaged over all experiments
(c) The fraction of solutions for which we obtained an integer solution by the LP execution (without any fixing and any rounding iterations)
(d) The number of "fixings" required to obtain integer solutions (without any rounding iterations), averaged over all experiments; this is the average number of fixing iterations performed to move from (c) to (e)
(e) The fraction of solutions that are integers after fixing iterations (without any rounding iterations)
(f) Average number of fixing and rounding iterations for the cases that we performed a single rounding iteration (this is the average number of fixing and rounding iterations performed to move from (e) to (g))
(g) The fraction of solutions that are integer after fixing and rounding iterations
(h) Average running time (in sec): the average running time of the simulation experiments, including the tableau creation, the LP (or ILP) execution and the fixing and rounding iterations until we obtain integer solutions

A. Perturbation improvements

Tables I and II present results for the random perturbation technique for load equal to 0.5 and 0.75. From these tables we can see that the random perturbation technique yields more integer solutions (metric (c)) when compared to the execution of the algorithm without it. Moreover, in some cases the algorithm without random perturbation was not able to find as good solutions with respect to the maximum number of wavelengths (metric (a)). As expected, the integrality performance deteriorates as the load increases. With respect to the examined cost functions, the linear cost function has good integrality performance for low load, but its performance deteriorates more rapidly than the other's two as the load increases. Note that, as metric (h) indicates, the improvements in integrality were obtained without a deterioration in the execution time. For the rest of the paper we will use the perturbation technique as it was proven to improve the integrality performance.

B. Comparing the formulations

In Tables III and IV we present performance results for the link, path, and decomposed (Multicommodity + Smallest Last WA) formulations, for load equal to 0.5 and 0.75 respectively. With respect to the formulations we can observe that the path formulation exhibits the best performance. The link formulation has more variables than the path formulation, as discussed in Section II.D. Although the link formulation has acceptable integrality performance (metric (c) and (e)) its execution takes longer time (metric (h)) than paths'. Note that the execution time of the link formulation is dominated by the tableau creation, due to the large number of variables and constraints. Moreover, the large number of variables results in spending more fixing and rounding iterations that the path (metric (f)), and thus there are cases that it finds solutions utilizing more wavelengths (metric (a)). As expected, the decomposed algorithm is much faster than the path and link formulations (metric (h)) but has worse wavelength performance (metric (a)).

With respect to the cost functions, we can see that the linear cost function is the best for the link formulation. However, when the path and decomposed formulations are considered, the square and the exponential cost functions exhibit the best

TABLE I

PERTURBATION RESULTS FOR 0.5 LOAD

Formulation	Cost Function	a	b	c	d	e	f	g	h
Path+ random perturbation	x	7.19	197.7	0.72	0.43	0.94	0.14	1	0.8
	x^2	7.18	198.9	0.78	0.23	0.96	0.31	1	1.1
	$2^{x/(w+1-x)}$	7.18	203.9	0.76	0.18	0.92	0.87	1	1.4
Path without perturbation	x	7.21	197.7	0.10	2.14	0.94	0.18	1	0.8
	x^2	7.2	198.6	0.06	1.55	0.90	0.59	1	1.1
	$2^{x/(w+1-x)}$	7.19	204.0	0.04	2.01	0.86	1.08	1	1.5

TABLE II

PERTURBATION RESULTS FOR 0.75 LOAD

Formulation	Cost Function	a	b	c	d	e	f	g	h
Path+ random perturbation	x	10.09	296.8	0.14	2.05	0.97	0.02	1	4.2
	x^2	10.09	298.4	0.37	1.23	0.98	0.22	1	4.5
	$2^{x/(w+1-x)}$	10.09	303.1	0.68	0.28	0.91	0.80	1	4.5
Path without perturbation	x	10.11	296.8	0	3.94	0.97	0.23	1	3.4
	x^2	10.09	297.5	0	3.17	0.94	0.43	1	5.4
	$2^{x/(w+1-x)}$	10.09	303.4	0	3.63	0.86	1.39	1	6.8

TABLE III

RESULTS FOR 0.5 LOAD

Formulation	Cost Function	a	b	c	d	e	f	g	h
Link	x	7.19	197.9	0.59	0.01	0.60	2.47	1	6.3
	x^2	7.2	200.1	0.67	0.02	0.69	2.40	1	8.6
	$2^{x/(w+1-x)}$	7.2	205.6	0.58	0.08	0.71	2.34	1	9.7
	min-max	7.23	222.4	0	1.96	0.28	8.54	1	4.4
Path	x	7.19	197.7	0.72	0.43	0.94	0.14	1	0.8
	x^2	7.18	198.9	0.78	0.23	0.96	0.31	1	1.1
	$2^{x/(w+1-x)}$	7.18	203.9	0.76	0.18	0.92	0.87	1	1.4
	min-max	7.22	218.6	0	2.45	0.28	10.74	1	1.0
Decomposed (MCF+SLWA)	x	7.33	196.9	0.99	0	0.99	0.02	1	0.1
	x^2	7.24	198.1	0.91	0.09	1	0	1	0.1
	$2^{x/(w+1-x)}$	7.22	203.8	0.86	0.08	0.94	0.22	1	0.1
	min-max	7.39	205.7	0.24	0.04	0.28	2.70	1	0.1
ILP Path	$2^{x/(w+1-x)}$	7.18	204.1	1	0	1	0	1	18.2

TABLE IV

RESULTS FOR 0.75 LOAD

Formulation	Cost Function	a	b	c	d	e	f	g	h
Link	x	10.09	298.6	0.19	0.85	0.98	0.02	1	14.1
	x^2	10.1	302.6	0.57	0.45	0.78	0.85	1	17.4
	$2^{x/(w+1-x)}$	10.09	304.5	0.55	0.09	0.62	1.57	1	20.5
	min-max	10.14	325.8	0	2.04	0.22	8.55	1	12.2
Path	x	10.09	296.8	0.14	2.05	0.97	0.02	1	4.2
	x^2	10.09	298.4	0.37	1.23	0.98	0.22	1	4.5
	$2^{x/(w+1-x)}$	10.09	303.1	0.68	0.28	0.91	0.80	1	4.5
	min-max	10.12	332.7	0	2.55	0.26	13.74	1	3.0
Decomposed (MCF+SLWA)	x	10.18	296.4	0.99	0.02	1	0	1	0.1
	x^2	10.11	297.4	0.95	0.05	1	0	1	0.1
	$2^{x/(w+1-x)}$	10.1	303.2	0.87	0.09	0.96	0.13	1	0.2
	min-max	10.28	306.9	0.24	0.05	0.29	2.74	1	0.1
ILP Path	$2^{x/(w+1-x)}$	10.09	303.3	1	0	1	0	1	97.7

performance. In the path formulation the square and exponential cost functions manage to find the same number of wavelengths as the ILP execution. The min-max cost function looses a few optimum solutions (when compared to the ILP) due to the large number of fixings and roundings that it performs (metric (f)). Moreover, when the min-max is compared to the linear and piecewise linear cost functions, the min-max utilizes a higher sum of wavelengths (metric (b)). Having a high number of utilized wavelengths can have negative effect on serving future connections. From this we can deduce that the linear and piecewise linear cost functions not only utilize fewer maximum wavelengths but also distribute the lightpaths in an efficient way so as to utilize fewer wavelengths throughout the network. Finally, from these tables we can observe that the path formulation when the square and exponential cost functions were utilized managed to find the same solutions with the ILP execution but with execution time at least one order of magnitude less. Higher gains in execution time were obtained for higher loads.

IV. CONCLUSIONS

We presented various algorithms for solving the static RWA problem with the objective of minimizing the maximum number of requested wavelengths based on LP relaxation formulations. The flow cost functions that are used in these formulations result in providing integer optimal solutions despite the absence of integrality constraints for a large subset of RWA input instances. We presented a random perturbation technique that was shown to increase the number of instances for which we find integer solutions. We also presented appropriate iterative fixing and rounding methods to be used when the algorithms do not yield integer solutions. Our results indicate that the proposed algorithms outperform the min-max formulation that is widely used in the literature and approximate the performance of an ILP execution with respect to the maximum number of required wavelengths, while decreasing the execution time by several orders of magnitude depending on the network topology and the traffic load.

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